The Navier-Stokes equations describe fluid motions in a wide range of physical circumstances and hence have applications in systems ranging from the oceans to the atmosphere. In spite of their importance, basic questions about the existence of smooth solutions of these equations in three-dimensional domains still exist. In this talk I will describe the physical origin of these questions and explain why the answers to them are so different in two and three dimensions, as well as why two-dimensional fluid mechanics is of interest in a three-dimensional world. (Received September 15, 2011)