A homotopy sphere is a manifold with the same homotopy groups as a sphere. The n-dimensional Poincare Conjecture states that an n-dimensional homotopy sphere is homeomorphic to the n-sphere. In the topological category, with the homeomorphisms only required to be continuous, the conjecture is now known to hold in all dimensions.

The smooth Poincare conjecture asks whether a smooth n-dimensional homotopy sphere is smoothly homeomorphic to the n-sphere. The answer in this case is known in all dimensions except four. It is still not known whether a smooth 4-manifold that is homotopy equivalent to a sphere is diffeomorphic to the 4-sphere.

This talk will given an overview of this conjecture, explain why techniques that work in other dimensions fail in dimension four, and discuss some possible approaches. (Received September 21, 2011)