Knots and links are a common theme in artworks that span cultures and time periods. While Celtic knots can be realized in a physical form, it is more common to see these knots as a surface decoration on paper or stone. The construction of Celtic knots by the Bain method uses a tessellation by squares where arcs connect vertex points of the squares. A new method of creating more complex knots and links is presented which uses a modular approach that begins with a $k$-uniform tessellation. Each regular polygon is decorated with a simple tangle that has arcs connecting uniformly spaced points on the sides of the polygons. A variety of complex knots and links can be created using this procedure. The use of a collection of tangles to comprise the knots and links allows one to use knot theory to analyze such knots and links. Examples of visually interesting knots and links created using this procedure are presented. (Received September 18, 2011)