Jonathan Sondow* (jsondow@alumni.princeton.edu). Evaluation of infinite products involving Fibonacci and Lucas numbers.

The Fibonacci numbers $F_k$ are defined by the recurrence $F_k = F_{k-1} + F_{k-2}$ with $F_0 = 0$ and $F_1 = 1$; the first few terms are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, . . . . The Lucas numbers $L_k$ satisfy the same recurrence, but with $L_0 = 2$ and $L_1 = 1$; the sequence begins 2, 1, 3, 4, 7, 11, 18, 29, 47, 76, . . . .

In this talk, I will begin by recalling formulas for $F_k$ and $L_k$ in terms of the golden ratio $\varphi = (1 + \sqrt{5})/2$. Then I will explain how to evaluate the infinite products

$$\prod_{n=1}^{\infty} \left(1 + \frac{1}{F_{2^n+1}}\right) = \frac{3}{2} \frac{6}{5} \frac{35}{34} \cdots = \frac{3}{\varphi}, \quad \prod_{n=1}^{\infty} \left(1 + \frac{1}{L_{2^n+1}}\right) = \frac{5}{4} \frac{12}{11} \frac{77}{76} \cdots = 3 - \varphi.$$

A preprint is available at http://arxiv.org/abs/1106.4246. (Received September 22, 2011)