Many of the biggest problems in additive number theory (such as Goldbach’s conjecture and Fermat’s last theorem) can be recast as understanding the behavior of sums of a set with itself. A sum-dominant set is a finite set \( A \subset \mathbb{Z} \) such that \(|A + A| > |A - A|\). It was initially believed that the percentage of subsets of \( \{0, \ldots, n\} \) that are sum-dominant tends to zero, however, in 2006 Martin and O’Bryant proved a positive percentage are sum-dominant.

We generalize their result to deal with many different ways of taking sums and differences of a set. We first prove that \(|\epsilon_1 A + \cdots + \epsilon_k A| > |\delta_1 A + \cdots + \delta_k A|\) a positive percent of the time for all nontrivial choices of \(\epsilon_j, \delta_j \in \{-1, 1\}\). Previous approaches proved the existence of many such sets given the existence of one; however, no method existed to construct such a set.

Extending this result, we find sets that exhibit different behavior as more sums/differences are taken. For example, we say \(A\) is \(k\)-generational if \(A, A + A, \ldots, kA\) are all sum-dominant. Numerical searches were unable to find even a 2-generational set, however, we prove that for any \(k\) a positive percentage of sets are \(k\)-generational, and no set can be \(k\)-generational for all \(k\). (Received September 14, 2011)