We give a criterion when an expansion of the ordered set of real numbers defines the image of \((\mathbb{R}, +, \cdot, \mathbb{N})\) under a semialgebraic injection. This allows us to answer several questions raised by Chris Miller about expansions of the additive group of real numbers.

**Theorem 1** ([1]). Let \(\alpha \in \mathbb{R}\) be a non-quadratic irrational number and let \(f : \mathbb{R} \to \mathbb{R}\) be the function that maps \(x\) to \(\alpha x\). Then \((\mathbb{R}, <, +, \mathbb{N}, f)\) defines multiplication on \(\mathbb{R}\).

**Theorem 2** ([1]). \((\mathbb{R}, <, +, \sin, \mathbb{N})\) defines multiplication on \(\mathbb{R}\).

This is joint with Michael Tychonievich.

**References**


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