Let $\lambda_1(G)$, $\lambda_2(G)$ and $\tau(G)$ denote the largest eigenvalue, the second largest eigenvalue and the maximum number of edge-disjoint spanning trees of a graph $G$, respectively. Motivated by a question of Seymour on the relationship between eigenvalues of a graph $G$ and bounds of $\tau(G)$, Cioabă and Wong conjectured that for any integers $d, k \geq 2$ and a $d$-regular graph $G$, if $\lambda_2(G) < d - \frac{2k-1}{d+1}$, then $\tau(G) \geq k$. They proved the conjecture for $k = 2, 3$, and presented evidence for the cases when $k \geq 4$. Thus the conjecture remains open for $k \geq 4$. We propose a more general conjecture that for a graph $G$ with minimum degree $\delta \geq 2k \geq 4$, if $\lambda_1(G) + \lambda_2(G) < 2\delta - \frac{2k-1}{\delta+1}$, then $\tau(G) \geq k$. We prove that for a graph $G$ with minimum degree $\delta$, each of the following holds.

(i) For $k \in \{2, 3\}$, if $\delta \geq 2k$ and $\lambda_1(G) + \lambda_2(G) < 2\delta - \frac{2k-1}{\delta+1}$, then $\tau(G) \geq k$.

(ii) For $k \geq 4$, if $\delta \geq 2k$ and $\lambda_1(G) + \lambda_2(G) < 2\delta - \frac{3k-1}{\delta+1}$, then $\tau(G) \geq k$. In particular, for a $d$-regular graph $G$ with $d \geq 2k \geq 4$, if $\lambda_2(G) < d - \frac{3k-1}{d+1}$, then $\tau(G) \geq k$. (Received September 20, 2012)