Parallelogram polyominoes, the sandpile model on $K_{m,n}$ and a $q,t$-Narayana polynomial.

In this talk I will highlight some results from a recent paper (arXiv:1208.0024) that was motived by a correspondence between bivincular patterns and composition matrices. We classify recurrent configurations of the sandpile model on the complete bipartite graph $K_{m,n}$ in terms of polyominoes. A canonical toppling process on these recurrent states gives rise to a bounce path within the corresponding polyomino. This bounce path, in turn, gives rise to a polynomial that we call the $q,t$-Narayana polynomial. We discuss this $q,t$-Narayana polynomial and its relation to the well-known $q,t$-Catalan polynomial. The specialization to the original correspondence between bivincular patterns and a sub-collection of recurrent configurations will also be touched upon. (Received September 21, 2012)