Game matching number of graphs.

We study a competitive optimization version of $\alpha'(G)$, the maximum size of a matching in a graph $G$. Players alternate adding edges of $G$ to a matching until it becomes a maximal matching. One player (Max) wants that matching to be large; the other (Min) wants it to be small. The resulting sizes under optimal play when Max or Min starts are denoted $\alpha'_g(G)$ and $\hat{\alpha}'_g(G)$, respectively. We show that always $|\alpha'_g(G) - \hat{\alpha}'_g(G)| \leq 1$. We obtain a sufficient condition for $\alpha'_g(G) = \alpha'(G)$ that is preserved under cartesian product. Always $\alpha'_g(G) \geq \frac{2}{3} \alpha'(G)$, with equality for many split graphs, while $\alpha'_g(G) \geq \frac{3}{4} \alpha'(G)$ when $G$ is a forest. Whenever $G$ is a 3-regular $n$-vertex connected graph, $\alpha'_g(G) \geq n/3$, and such graphs exist with $\alpha'_g(G) \leq 7n/18$. For an $n$-vertex path or cycle, the value is roughly $n/7$. (Received September 23, 2012)