The set of all permutations \( \{1, 2, \ldots, n\} \), for any \( n \), forms a poset, where the relation is pattern containment. We study the Möbius function on the intervals of this poset. This seems to be hard for generic intervals. However, there is a computationally effective solution for the separable permutations, which are those that can be decomposed as sums and skew sums of singletons, equivalently, those permutations that avoid 2413 and 3142. There is also an effective formula for reducing the computation of the Möbius function on intervals of decomposable permutations to its computation for indecomposable ones. For indecomposable permutations, though, there are no general formulas yet, so that’s the brick wall we are currently pounding on.

The Möbius function of an interval \( \mathcal{I} \) is equivalent to the Euler characteristic of the simplicial complex consisting of the chains of \( \mathcal{I} \). We would like to understand more of the topology of these intervals. Here we know almost nothing, although there are indications that there may be many cases where the topology is nice and simple, such as being homotopy equivalent to a wedge of spheres.

Joint work with B. Tenner and with V. Jelínek, E. Jelinková and A. Burstein, and (in progress) with P. McNamara. (Received September 24, 2012)