Greg Kuperberg* (greg@math.ucdavis.edu), Shachar Lovett (slovett@math.ias.edu) and Ron Peled (peledron@post.tau.ac.il). Probabilistic existence of combinatorial and geometric $t$-designs.

A $t$-design on an affine real algebraic variety with a measure is a finite set whose moments up to degree $t$ match those of the measure on the whole space. This definition includes both combinatorial $t$-designs (sets of $k$-subsets of a $v$-set that cover each $t$-set the same number of times) and geometric $t$-designs, which includes $t$-designs on a sphere surface.

Two fundamental theorems, Tierlinck’s theorem in the combinatorial case and the Seymour-Zaslavsky theorem in the spherical case, say simply that a non-trivial $t$-design exists for every $t$. These results were originally proved constructively, but with poor or highly restricted asymptotics.

We discuss a new approach to both theorems based on simply picking a set of points at random. The chance that the set is a $t$-design is very small, but in a favorable regime it can be estimated and is positive. In the geometric case the probability of success is exactly zero, but the probability density is positive, which suffices. (Received August 24, 2012)