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**Alexandr Kostochka** and **Matthew Yancey\*** (yancey1@illinois.edu). *Ore's Conjecture on color-critical graphs is almost true.*

A graph  $G$  is  $k$ -critical if it has chromatic number  $k$ , but every proper subgraph of  $G$  is  $(k - 1)$ -colorable. Let  $f_k(n)$  denote the minimum number of edges in an  $n$ -vertex  $k$ -critical graph. We give a lower bound,  $f_k(n) \geq F(k, n)$ , that is sharp for every  $n = 1 \pmod{k - 1}$ . It is also sharp for  $k = 4$  and every  $n \geq 6$ . The result improves the classical bounds by Gallai and Dirac and subsequent bounds by Krivelevich and Kostochka and Stiebitz. It establishes the asymptotics of  $f_k(n)$  for every fixed  $k$ . It also proves that the conjecture by Ore from 1967 that for every  $k \geq 4$  and  $n \geq k + 2$ ,  $f_k(n + k - 1) = f_k(n) + \frac{k-1}{2}(k - \frac{2}{k-1})$  holds for each  $k \geq 4$  for all but at most  $k^3/12$  values of  $n$ . We will also characterize all  $k$ -critical graphs for which  $|E(G)| = F(k, |V(G)|)$ . (Received September 04, 2012)