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In 1976 Carlitz, Scoville, and Vaughan proved a simple, but fundamental result about the enumeration of words with restrictions on adjacent letters:

Let A be an alphabet and let R be a relation on A , that is, a subset of $A \times A$. Let A^R be the set of all words $a_1 a_2 \cdots a_k$ on A , for all k , with $a_j a_{j+1} \in R$ for $1 \leq j \leq k - 1$. (Thus the empty word and all one-letter words are in A^R .) Let $\bar{R} = A \times A - R$. Then the Carlitz-Scoville-Vaughan theorem asserts that

$$\sum_{w \in A^R} w = \left(\sum_{w \in A^{\bar{R}}} (-1)^{|w|} w \right)^{-1},$$

where $|w|$ is the length of w .

The Carlitz-Scoville-Vaughan theorem deserves to be much better known than it is. I will explain how it can be applied to many interesting permutation enumeration problems, such as counting permutations with periodic run lengths and counting pairs of permutations with no common descents. (Received September 09, 2012)