Inspired by a recent work of Björner and Vorwerk, we introduce a notion of $i$-banner complexes: for various values of $i$ these complexes interpolate between the class of flag complexes and the class of all simplicial complexes. We provide examples of simplicial spheres of an arbitrary dimension that are $(i+1)$-banner, but not $i$-banner. We then establish $i$-banner analogues of several theorems for flag complexes. For instance, we prove that (1) the codimension-$(i+j-1)$ skeleton of an $i$-banner homology sphere $\Delta$ is $2(i+j)$-Cohen–Macaulay for all $0 \leq j \leq \dim \Delta + 1 - i$, and that (2) for every $i$-banner simplicial complex $\Delta$ there exists a balanced complex $\Gamma$ whose face numbers of dimension $i-1$ and higher coincide with those of $\Delta$. (Received September 12, 2012)