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Constructing Graphs with No Immersion of Large Complete Graphs.

In 2003, Abu-Khazam and Langston conjectured that every d -chromatic graph contains an immersion of K_d . Lescure and Meyniel proved the conjecture for $d = 5, 6$ and DeVos, Kawarabayashi, Mohar, and Okamura proved the conjecture for $d = 5, 6, 7$. Both proofs use the fact that a d -critical graph has minimum degree at least $d - 1$. Seymour and later DeVos, Dvořák, Fox, McDonald, Mohar, and Scheide showed that the stronger conjecture that a graph with minimum degree $d - 1$ has an immersion of K_d fails for $d = 10$ and $d \geq 12$, but it is shown by DeVos et al. that a minimum degree of $200d$ does guarantee an immersion of K_d .

Here we will show that the stronger conjecture is false for $d \geq 8$ and give infinite families of examples with minimum degree $d - 1$ and chromatic number $d - 2$ or $d - 1$ that do not contain an immersion of K_d . We will give examples that can be up to $(d - 2)$ -edge-connected. We show, using Hajós' Construction, that there is an infinite class of non- $(d - 1)$ -colorable graphs that contain an immersion of K_d . We also make a conjecture for the minimum number of vertices that must have degree at least md , m a positive integer, to guarantee an immersion of K_d . (Received July 11, 2012)