Cluster algebras are commutative rings with a set of distinguished generators having a remarkable combinatorial structure. They were introduced by Fomin and Zelevinsky in 2000 in the context of representation theory, but have since appeared in many other contexts, from Poisson geometry to triangulations of surfaces and Teichmuller theory.

In this talk I will give a gentle introduction to cluster algebras and then sketch how the theory led to a solution of Zamolodchikov’s periodicity conjecture in mathematical physics. (Received September 16, 2012)