Jensen’s formula relates the Mahler measure of a one-variable polynomial to a simple formula depending on the roots of the polynomial. The situation for multivariate polynomials is quite different. Beginning in the 1970s and continuing to the present day, researchers have explored the connection between the Mahler measure of a polynomial defining an elliptic curve and the $L$-function of that curve. A natural extension of this line of inquiry involves connecting the polynomials whose zeros define Calabi-Yau varieties of dimension greater than one with $L$-functions associated to those varieties. In two dimensions, that means connecting the Mahler measure of polynomials to the $L$-functions of elliptic $K3$-surfaces.

Building on previous work of Bertin, we prove three new formulas of this type. The strategy for proving these formulas is as follows:

- Understand the transcendental lattice and the group of sections for the $K3$-surface.
- Relate the Mahler measure of the polynomial to the $L$-function of a modular form.
- Relate the $L$-function of the $K3$-surface to the $L$-function of that same modular form.

We will outline each piece of the argument and point out technical difficulties that arise in some cases. (Received September 18, 2012)