Inverse spectral geometry asks the extent to which the topology and geometry of a Riemannian manifold $M$ is determined by its spectrum with respect to the Laplace-Beltrami operator. Whereas volume and scalar curvature are spectral invariants, the isometry class is not. In this talk we will review several constructions of isospectral but not isometric Riemannian manifolds which are lattice-theoretic in nature. We will then construct, using the arithmetic of orders in quaternion algebras, examples of isospectral but not isometric hyperbolic surfaces which have extremely small volume. These examples have minimal volume among all isospectral surfaces arising from maximal arithmetic Fuchsian groups. This is joint work with Peter Doyle and John Voight. (Received July 20, 2012)