Correlations of Fractional Parts of Dilated Harmonic Sequences.

The harmonic sequence is $y_k = 1/k$, and for positive integer $n$ let $x_k = \{n/k\}$ be the fractional parts of the dilated harmonic sequence $n/k$. We consider the distribution of the fractional parts of the initial part of the sequence $x_k$ from 1 to $f(n)$, where we will let $f(n) \to \infty$ as $n \to \infty$. For example, taking $f(n) = n$ it is known that the average value of the fractional parts is $1 - \gamma = 0.42278\ldots$, where $\gamma$ is Euler’s constant, a result of de la Vallee Poussin. We study statistics attached to such distributions, including all the $r$-point distributions of $(x_k, x_{k+1}, \ldots, x_{k+r})$. We determine sufficient conditions on $f(n)$ to get a limiting distribution, and determine information about this distribution given in terms of its Fourier coefficients, which depend on the function $f(n)$. (Received September 22, 2012)