Let $\ell$ be a prime, and let $E/\mathbb{Q}$ be an elliptic curve. The action of the absolute Galois group $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on the $\ell$-torsion subgroup $E[\ell]$ induces a group representation $\rho_{E,\ell}: \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \text{Aut}(E[\ell]) \simeq \text{GL}_2(\mathbb{Z}/\ell\mathbb{Z})$. A conjecture of Serre states that there is an absolute bound $\ell_{\text{max}}$ such that $\rho_{\ell,E}$ is surjective for all primes $\ell > \ell_{\text{max}}$ and all elliptic curves $E/\mathbb{Q}$ without complex multiplication (CM); it is generally believed that the conjecture holds with $\ell_{\text{max}} = 37$. This implies that there is a finite set of groups that arise as the image of a non-surjective representation $\rho_{\ell,E}$ for an elliptic curve $E/\mathbb{Q}$ without CM. As a first step toward computing this set, I will describe a highly efficient algorithm for computing $\rho_{\ell,E}$ (up to isomorphism and usually up to conjugacy) for all primes $\ell$ up to a given bound $\ell_{\text{max}}$ and all elliptic curves $E$ in a given family. I will then present results covering all the elliptic curves without CM listed in Cremona’s tables or the Stein-Watkins database. (Received September 24, 2012)