Conway and Smith, in their book “On Quaternions and Octonions”, studied factorization for integral (Hurwitz) quater-
nions, and introduced the metacommutation problem. Namely, if $P$ and $Q$ are prime Hurwitz integers of norms $p$ and $q$ (say) respectively, then $u = PQ$ also has a factorization of the form $u = Q'P'$, with $Q'$ and $P'$ of norms $q$ and $p$ respectively, unique up to unit-migration. They showed that factorization for the Hurwitz integers is unique up to the operations of unit-migration, metacommutation and recombination. The authors state that the problem of determining the metacommuting pair $(Q', P')$ given $(P, Q)$ does not seem to have been addressed in the literature. We study this question, and analyze some interesting properties of the metacommutation map. (Received September 24, 2012)