1086-11-2024 **Nathan Kaplan*** (nkaplan@math.harvard.edu) and **Noam Elkies** (elkies@math.harvard.edu). *Numerical Semigroups, t-core Partitions, and Weighted Theta Functions.*

A numerical semigroup is an additive submonoid S of $\mathbb{N}_0$ with finite complement. The size of the complement is the genus of S, $g(S)$, and the sum of the elements of the complement minus $g(S)(g(S)+1)/2$ is the weight of S, $w(S)$. We use the theory of modular forms, specifically weighted theta functions, to show that for all $n \geq 6$ there is a numerical semigroup S with smallest nonzero element 5 and $w(S)+g(S)=n$.

This problem is motivated by the theory of t-core partitions, partitions with no hook lengths divisible by t. A theorem of Granville and Ono says that for any $t \geq 4$ and $n \geq 1$ there exists a t-core partition of n. Given a semigroup S with smallest nonzero element t we construct a t-core partition from it of size $w(S)+g(S)$. We express this quantity as a quadratic function in $t-1$ variables. For $t=5$, we study this function in detail and prove a stronger version of this result: for every $n \geq 6$ there exists a 5-core partition of n coming from a semigroup.

For $t=5$ this function leads to a quadratic form related to the A_4 lattice. The condition that the inputs come from a semigroup leads us to restrict to inputs in a certain cone. We then study integers represented by these vectors using weighted theta functions. (Received September 24, 2012)