A numerical semigroup is an additive submonoid \( S \) of \( \mathbb{N}_0 \) with finite complement. The size of the complement is the genus of \( S \), \( g(S) \), and the sum of the elements of the complement minus \( g(S)(g(S) + 1)/2 \) is the weight of \( S \), \( w(S) \). We use the theory of modular forms, specifically weighted theta functions, to show that for all \( n \geq 6 \) there is a numerical semigroup \( S \) with smallest nonzero element 5 and \( w(S) + g(S) = n \).

This problem is motivated by the theory of \( t \)-core partitions, partitions with no hook lengths divisible by \( t \). A theorem of Granville and Ono says that for any \( t \geq 4 \) and \( n \geq 1 \) there exists a \( t \)-core partition of \( n \). Given a semigroup \( S \) with smallest nonzero element \( t \) we construct a \( t \)-core partition from it of size \( w(S) + g(S) \). We express this quantity as a quadratic function in \( t - 1 \) variables. For \( t = 5 \), we study this function in detail and prove a stronger version of this result: for every \( n \geq 6 \) there exists a 5-core partition of \( n \) coming from a semigroup.

For \( t = 5 \) this function leads to a quadratic form related to the \( A_4 \) lattice. The condition that the inputs come from a semigroup leads us to restrict to inputs in a certain cone. We then study integers represented by these vectors using weighted theta functions. (Received September 24, 2012)