Let $N$ be an arbitrary positive integer. We consider continued fractions of the form

$$a_0 + \frac{N}{a_1 + \frac{N}{a_2 + \frac{N}{a_3 + \cdots}}}$$

with $a_0$ a nonnegative integer and $a_1, a_2, a_3, \ldots$ positive integers, and refer to them as $\text{cf}_N$ expansions.

The $N > 1$ case has both a number of similarities to and some surprising differences from the classical, i.e., $N = 1$, case.

For $N > 1$, every positive real number $x_0$ has infinitely many $\text{cf}_N$ expansions. We develop a natural notion of the best $\text{cf}_N$ expansion of $x_0$.

We show, for example, that for $N > 1$, every quadratic irrationality has both periodic and nonperiodic $\text{cf}_N$ expansions, and that in many cases the best $\text{cf}_N$ expansion of a quadratic irrationality is periodic, but, on the grounds of extensive computational results, we conjecture that this is not always the case. We establish further results about the form of the best $\text{cf}_N$ expansion when it is periodic; this form sometimes but not always more or less resembles the form in the classical case.

In the classical case, continued fractions have a close relationship with Pell’s equation, and we investigate the analog for $N > 1$ as well. (Received May 10, 2012)