I will describe a speculative project to enumerate every elliptic curve over the field $\mathbb{Q}(\sqrt{5})$, up to the first curve of rank 4. We use an efficient implementation of an algorithm of Dembele and fast sparse linear algebra to compute tables of Hilbert modular forms of weight $(2,2)$ over $\mathbb{Q}(\sqrt{5})$. Then, via a variety of methods, we construct the corresponding elliptic curves. To have any hope to someday reach our far-off goal, the implementations much be highly optimized; moreover, just keeping track of the enormous amount of data we generate is challenging. This is joint work with Jonathan Bober, Alyson Deines, Ariah Klages-Mundt, Benjamin LeVeque, R. Andrew Ohana, Sebastian Pancratz, Ashwath Rabindranath, Paul Sharaba, and Christelle Vincent. (Received September 25, 2012)