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Gronwall's function G is defined for $n > 1$ by $G(n) = \frac{\sigma(n)}{n \log \log n}$ where $\sigma(n)$ is the sum of the divisors of n . We call an integer $N > 1$ a *GA1 number* if N is composite and $G(N) \geq G(N/p)$ for all prime factors p of N . We say that N is a *GA2 number* if $G(N) \geq G(aN)$ for all multiples aN of N . In "Robin's theorem, primes, and a new elementary reformulation of the Riemann Hypothesis," we used Robin's and Gronwall's theorems on G to prove that the Riemann Hypothesis (RH) is true if and only if 4 is the only number that is both GA1 and GA2. In the present paper, we study GA1 numbers and GA2 numbers separately. We compare them with superabundant (SA) and colossally abundant (CA) numbers (first studied by Ramanujan). We give algorithms for computing GA1 numbers; the smallest one with more than two prime factors is 183783600, while the smallest odd one is 1058462574572984015114271643676625. We find nineteen GA2 numbers ≤ 5040 , and prove that a GA2 number $N > 5040$ exists if and only if RH is false, in which case N is even and $> 10^{8576}$. Our paper is to appear in the Ramanujan Journal. (Received September 26, 2012)