Let $G = \mathbb{Z}/n\mathbb{Z}^*$ be the group of units of the ring $(\mathbb{Z}/n\mathbb{Z}, +n, \cdot n)$, and suppose that $f$ is a polynomial with integer coefficients. We explore the orbits under $f$, and ask if any algebraic structure is contained in such orbits. In particular:

When is the orbit of 1 under $f$ a cycle? If it is a cycle, do its elements form a subgroup of $G$? In this case, what algebraic structure is seen in this orbit and other orbits? When $f$ is a product of more than one cycle, the orbit of 1 may coincide with a (proper) subgroup $H$ of $G$. When this occurs, there is a natural, yet varied correspondence between the cosets of $H$ and the cycles of $f$. And finally, when we form a conjugate of such an $f$ by another bijection $g$ from $G$ to $G$, the algebraic structure of the orbit of 1 under $f$ is sometimes altered. (Received August 14, 2012)