Let $E$ be an elliptic curve over $\mathbb{Q}$. For each prime $p$ of good reduction, $E$ reduces to a curve $E_p$ over the finite field $\mathbb{F}_p$ with $\#E_p(\mathbb{F}_p) = p + 1 - a_p$, where $|a_p(E)| \leq 2\sqrt{p}$. In this talk, we discuss the problem of determining how often $\#E(\mathbb{F}_p)$ is squarefree. Our results in this vein are twofold. For any fixed curve $E$, we give an asymptotic formula for the number of primes up to $X$ for which $\#E_p(\mathbb{F}_p)$ is squarefree. This resolves affirmatively a conjecture of David and Urroz. Moreover, we use sieve methods to improve upon a result of Gekeler that computes the average number of primes up to $X$ for which $\#E_p(\mathbb{F}_p)$ is squarefree (over curves $E$ in a suitable box). This talk is based on joint work with Shabnam Akhtari, Chantal David, and Heekyoung Hahn. (Received August 15, 2012)