1086-11-296 **Andrew Bremner*** (bremner@asu.edu). *Arithmetic progressions of squares in cubic fields.*

Euler showed that the length of the longest arithmetic progression (AP) of integer squares is equal to three, for example, 1, 25, 49. Recently, Xarles (2011) investigated APs in number fields, and proved the existence of an upper bound $K(d)$ for the maximal length of an AP of squares in a number field of degree d. He shows that $K(2) = 5$, with example 7^2, 13^2, 17^2, $\sqrt{409}^2$, 23^2. Cubic fields present difficulty, and $K(3)$ is unknown. There are infinitely many examples of cubic fields containing APs of length 4, but no cubic field is known with a non-trivial AP of length 5. We show why it is very unlikely such a field exists. Cubic fields with such an AP are parametrized by the rational points on three rational curves of genus 8, 9, 9, so by Faltings Theorem, are finite in number. We have been unable to determine the rational points on these curves, but search indicates only trivial solutions. (Received August 16, 2012)