What is the probability that a random integer is squarefree? Prime? How many number fields of degree d are there with discriminant at most X? What does the class group of a random quadratic field look like? These questions, and many more like them, are part of the very active subject of arithmetic statistics. Many aspects of the subject are well-understood, but many more remain the subject of conjectures, by Cohen-Lenstra, Malle, Bhargava, Batyrev-Manin, and others.

In this talk, I explain what arithmetic statistics looks like when we start from the field $\mathbb{F}_q(x)$ of rational functions over a finite field instead of the field $\mathbb{Q}$ of rational numbers. The analogy between function fields and number fields has been a rich source of insights throughout the modern history of number theory. In this setting, the analogy reveals a surprising relationship between conjectures in number theory and conjectures in topology about stable cohomology of moduli spaces, especially spaces related to Artin’s braid group. I will discuss some recent work in this area, in which new theorems about the topology of moduli spaces lead to proofs of arithmetic conjectures over function fields, and to new, topologically motivated questions about counting arithmetic objects. (Received September 24, 2012)