Let $R$ be a normed domain with fraction field $K$. Let $D$ be a finite-dimensional alternative $K$-algebra, let $A$ be an $R$-order in $D$, and let $I$ be an ideal in $R$. Following H. Lenstra, we introduce the concept of $I$ being a Euclidean ideal. An ideal is Euclidean iff a certain associated norm form is a Euclidean form. When $D$ is a quadratic algebra (e.g. a quaternion or octonion algebra), we get a Euclidean quadratic form in the sense of our previous work.

When $R$ is a Dedekind domain, Lenstra showed that the existence of a Euclidean ideal forces the Picard group of $R$ to be cyclic; he also showed (e.g.) that if the ring of integers of a quadratic field admits a Euclidean ideal, then its Picard group has order at most 2. We present a non-commutative (but associative!) analogue of these results: in particular, under suitable hypotheses, if a maximal order in a quaternion algebra admits a Euclidean ideal it has class number at most 2. We use this result to rederive Fitzgerald’s classification of Euclidean ideals in definite quaternion orders over $\mathbb{Z}$. (Received September 05, 2012)