1086-11-494 **Pete L. Clark*** (pete@math.uga.edu). *Euclidean Ideals and Euclidean Forms.*

Let R be a normed domain with fraction field K. Let D be a finite-dimensional alternative K-algebra, let A be an R-order in D, and let I be an ideal in R. Following H. Lenstra, we introduce the concept of I being a **Euclidean ideal**. An ideal is Euclidean iff a certain associated norm form is a **Euclidean form**. When D is a quadratic algebra (e.g. a quaternion or octonion algebra), we get a Euclidean quadratic form in the sense of our previous work.

When R is a Dedekind domain, Lenstra showed that the existence of a Euclidean ideal forces the Picard group of R to be cyclic; he also showed (e.g.) that if the ring of integers of a quadratic field admits a Euclidean ideal, then its Picard group has order at most 2. We present a non-commutative (but associative!) analogue of these results: in particular, under suitable hypotheses, if a maximal order in a quaternion algebra admits a Euclidean ideal it has class number at most 2. We use this result to rederive Fitzgerald's classification of Euclidean ideals in definite quaternion orders over Z. (Received September 05, 2012)