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Carl B. Pomerance (carlp@math.dartmouth.edu), 6188 Kemeny Hall, Dartmouth College, Hanover, NH 03755, and **Hee-Sung Yang*** (hee-sung.yang.12@dartmouth.edu), 6188 Kemeny Hall, Dartmouth College, Hanover, NH 03755. *Variant of a theorem of Erdős on the sum-of-proper-divisors function.*

In 1973, Erdős proved that the upper density of the set $s(\mathbb{N})$ is less than 1, where $s(n) := \sigma(n) - n$ is the sum of the proper divisors of n . We investigate the analogous question where σ is replaced with similar divisor functions, such as the sum-of-unitary-divisors function σ^* (which sums those divisors d of n co-prime to n/d). We use a modified version of Erdős's original argument from the aforementioned work to prove that the upper density of $s^*(\mathbb{N})$ is less than 1, thereby showing that there are infinitely many integers not in the image of s^* . In one of the cases, the theory of covering congruences makes a surprising appearance. We also present an algorithm that allows us to enumerate the total number of integers not in $s^*(\mathbb{N})$ up to 10^8 (the previous known result, by David Wilson in 2001, was up to 10^5) and conjecture the density of the set $s^*(\mathbb{N})$ based on this result. (Received July 19, 2012)