Carl B. Pomerance (carlp@math.dartmouth.edu), 6188 Kemeny Hall, Dartmouth College, Hanover, NH 03755, and Hee-Sung Yang* (hee-sung.yang.12@dartmouth.edu), 6188 Kemeny Hall, Dartmouth College, Hanover, NH 03755. Variant of a theorem of Erdős on the sum-of-proper-divisors function.

In 1973, Erdős proved that the upper density of the set \( s(\mathbb{N}) \) is less than 1, where \( s(n) := \sigma(n) - n \) is the sum of the proper divisors of \( n \). We investigate the analogous question where \( \sigma \) is replaced with similar divisor functions, such as the sum-of-unitary-divisors function \( \sigma^* \) (which sums those divisors \( d \) of \( n \) co-prime to \( n/d \)). We use a modified version of Erdős’s original argument from the aforementioned work to prove that the upper density of \( s^*(\mathbb{N}) \) is less than 1, thereby showing that there are infinitely many integers not in the image of \( s^* \). In one of the cases, the theory of covering congruences makes a surprising appearance. We also present an algorithm that allows us to enumerate the total number of integers not in \( s^*(\mathbb{N}) \) up to \( 10^8 \) (the previous known result, by David Wilson in 2001, was up to \( 10^5 \)) and conjecture the density of the set \( s^*(\mathbb{N}) \) based on this result. (Received July 19, 2012)