The Burgess inequality and the least k-th power non-residue.

The Burgess inequality is the best upper bound we have for the character sum $S_{\chi}(M, N) = \sum_{M < n \leq M + N} \chi(n)$. Until recently, no explicit estimates had been given for the inequality. In 2006, Booker gave an explicit estimate for quadratic characters which he used to calculate the class number of a 32-digit discriminant. McGown used an explicit estimate to show that there are no norm-Euclidean Galois cubic fields with conductor greater than $10^{70}$. Both of their explicit estimates are on restricted ranges. In this talk we give an explicit estimate that works for any integers $M$ and $N$. We also improve McGown’s estimates in a slightly narrower range, getting explicit estimates for characters of any order. We apply the estimates to the question of how large must a prime $p$ be to ensure that there is a $k$-th power nonresidue less than $p^{1/6}$. (Received September 24, 2012)