Reduced $\tau_n$-factorizations of the Integers. Preliminary report.

Given a natural number $n$, a reduced $\tau_n$-factorization of an integer $a$ is a factorization of the type

$$a = a_1a_2 \ldots a_k,$$

where $a_1 \equiv a_2 \equiv \ldots \equiv a_k \mod n$ and $a_i \neq \pm 1$ for all $1 \leq i \leq k$. With these generalized factorizations new irreducible elements emerge. For example, for $n \geq 2$, $6 = 2 \cdot 3$ has no nontrivial reduced $\tau_n$-factorizations. The analogue of the Fundamental Theorem of Arithmetic, that any positive integer has a unique reduced $\tau_n$-factorization into these new irreducibles, fails in the existence part for most $n$. For the remaining $n$, the uniqueness of the factorization is not guaranteed. (Received September 24, 2012)