We study quadric surface bundles $X \to S$, i.e., families of quadric surfaces parameterized by $S$ and degenerating to cones along a divisor $D \subset S$. We show that when $S$ is a regular surface, quadric surface bundles are classified by quaternion Azumaya algebras over a double cover $T \to S$ branched along the degeneration divisor $D$. Among the many arithmetic and geometric applications of this result, in this talk we will highlight: constructing counterexamples to the local-global principle for zeros of quadratic forms over function fields of surfaces, and the geometry of cubic fourfolds containing a plane. (Received September 19, 2012)