Let an $n$-by-$n$ matrix $A$ be almost normal in a sense that it has $n - 1$ orthogonal eigenvectors. The properties of its numerical range $W(A)$ and Aluthge transform $\Delta$ are explored. In particular, it is proven that for unitarily irreducible almost normal $A$, $W(A)$ cannot have flat portions on the boundary and $\Delta(A)$ is not normal (the latter, under the additional conditions that $n > 2$ and $A$ is invertible). In passing, the unitary irreducibility criterion for $A$, almost normality criterion for $A^*$, and the rank formula for the self-commutator $A^*A - AA^*$ are established. (Received September 23, 2012)