Dominique Guillot* (dguillot@stanford.edu), Apoorva Khare (khare@stanford.edu) and Bala Rajaratnam (brajarat@stanford.edu). Functions operating on sparse positive definite matrices encoded by graphs.

We characterize entrywise real functions of matrices leaving invariant the cone of positive semidefinite matrices with prescribed structure of zeros encoded by a graph. This extends classical and important results by Rudin, Schoenberg, and others, in the case of complete graphs. Our results show in many cases that such functions are necessarily analytic with positive coefficients. The techniques used involve an interesting interplay between complex analysis, spectral theory, and harmonic analysis. In addition to the inherent theoretical interest, this problem has important consequences in machine learning (via kernels), in regularization of covariance matrices, and in the study of Toeplitz operators. (Received September 25, 2012)