We study the dual groups of a collection of metrizable group topologies for $\mathbb{R}^n$ that are weaker than the usual topology. These topologies are defined by choosing a sequence $\{v_i\}$ in $\mathbb{R}^n$ and specifying the approximate rate at which it converges to zero. If $\{v_i\}$ goes to infinity sufficiently fast in the usual topology, then such a group topology $T$ always exists. We prove that the group of continuous homomorphisms of $(\mathbb{R}^n, T)$ into the circle group is an uncountable subgroup of $\mathbb{R}^n$ that is dense in $\mathbb{R}^n$ in the usual topology, and its complement is also uncountable and dense. Since neither $(\mathbb{R}^n, T)$ nor its completion is locally compact, classical duality theory does not apply. (Received September 25, 2012)