Let $G$ be a Polish (i.e., complete separable metric topological) group. Define $G$ to be an algebraically determined Polish group if given any Polish group $L$ and an algebraic isomorphism $\varphi : L \to G$, then $\varphi$ is a topological isomorphism. The purpose of this paper is to prove a general theorem that gives useful sufficient conditions for a semidirect product of two Polish groups to be algebraically determined. This general theorem will provide a flowchart or recipe for proving that some special semidirect products are algebraically determined. For example, it may be used to prove that the natural semidirect product of $H$ and $G$, where $H$ is the additive group of a separable Hilbert space and $G$ is a Polish group of unitaries on $H$ acting transitively on the unit sphere with $-I \in G$, is algebraically determined. An example of such a $G$ is the unitary group of a separable irreducible $C^*$-algebra with identity on $H$. Not all nontrivial semidirect products of Polish groups are algebraically determined, for it is known that the Heisenberg group is not an algebraically determined Polish group. (Received June 26, 2012)