Given two sufficiently regular probability measures $\mu$ and $\nu$ on $\mathbb{R}^n$, general results due to Brenier (1987) guarantee the existence and uniqueness of an optimal transportation map between them, that is a map $T : \mathbb{R}^n \to \mathbb{R}^n$ such that $T_\# \mu = \nu$ and the quadratic cost $\int_{\mathbb{R}^n} |T(x) - x|^2 d\mu(x)$ is minimized. However, in general it is hard to say much about the map, let alone give an explicit expression for it. We present an efficient computer algorithm that uses a discretization of the problem to give perhaps the first pictures and movies of optimal transportation plans, for domains in $\mathbb{R}^2$. These suggest subtle relations between the geometry of the support of $\mu$ and $\nu$ and the regularity of $T$, some of which we are able to prove. (Received September 05, 2012)