In a recent paper, Tolsa has characterized $d$-regular uniformly rectifiable measures in Euclidean space using Wasserstein distances. For a $d$-regular measure $\mu$, he defines a quantity $\alpha(x,r)$ which, roughly speaking, measures the Wasserstein distance between $\mu$ inside the ball $B(x,r)$ and planar $d$-dimensional measure and proves that uniform rectifiability of $\mu$ is equivalent to $\alpha(x,r)^2 \frac{d\mu(x)dr}{r}$ being a Carleson measure. In this talk, we explore what conditions on $\alpha(x,r)$ are necessary to guarantee different grades of rectifiability for $\mu$ if we only assume $\mu$ is a doubling measure. We also establish rectifiability using more intrinsic quantities similar to $\alpha(x,r)$ involving the Wasserstein distance which estimate the doubling behavior of $\mu$. (Received September 16, 2012)