The purpose of this paper is to provide global existence and uniqueness results for a family of fluid transport equations by establishing convergence results for the particle method applied to these equations. The considered family of PDEs is a collection of strongly nonlinear equations which yield traveling wave solutions and can be used to model a variety of flows in fluid dynamics. We apply a particle method to the studied evolutionary equations and provide a new self-contained method for proving its convergence. The latter is accomplished by using the concept of space-time bounded variation and the associated compactness properties. From this result, we prove the existence of a unique global weak solution in some special cases and obtain stronger regularity properties of the solution than previously established. (Received September 21, 2012)