Otis Chodosh, Vishesh Jain (visheshj@stanford.edu), Michael Lindsey, Lyuboslav Panchev* (lpanchev@stanford.edu) and Yanir A. Rubinstein. On the discontinuity of optimal transportation maps.

Consider two bounded domains $\Omega$ and $\Lambda$ in $\mathbb{R}^2$, and two sufficiently regular probability measures $\mu$ and $\nu$ supported on them. By Brenier’s theorem, there exists a unique optimal transportation map $T$ satisfying $T_\#\mu = \nu$ and minimizing the quadratic cost $\int_{\mathbb{R}^2} |T(x) - x|^2 d\mu(x)$. Furthermore, by Caffarelli’s regularity theory for the real Monge–Ampère equations (1990’s) if $\Lambda$ is convex, $T$ is continuous. We study the converse problem, namely: how discontinuous is the map $T$ when $\Lambda$ fails to be convex? We prove a number of results relating the geometry of $\Lambda$ to the (dis)continuity of $T$. The main idea is to use tools of convex analysis and the extrinsic geometry of $\partial \Lambda$ to distinguish between Brenier and Alexandrov weak solutions of the Monge–Ampère equation. (Received September 05, 2012)