The measure of maximal entropy for a complex rational function varies continuously as one moves around in the parameter space of functions of fixed degree $d$. What happens as one approaches the boundary of this parameter space? By transferring this question to the Berkovich projective line, we answer it under a natural stability hypothesis. Our technique also gives a Markov process interpretation of equidistribution of pre-images for the limit measure, which allows one to explicitly determine the measure in many cases. (Received September 25, 2012)