We will discuss complex surface automorphisms with positive entropy, addressing in particular the question of what values of entropy can be achieved by such automorphisms. A fundamental result in this area is that any positive value of entropy is necessarily the logarithm of the entropy of a Salem number. However, a Salem number in general need not necessarily give the entropy of some automorphism. We will present a complete characterization of the entropies of automorphisms of two-dimensional complex tori, as well as recent progress in the characterization of entropies of automorphisms of abelian surfaces. We will also review results and open questions for automorphisms of rational surfaces and K3 surfaces. (Received September 09, 2012)