A $C^*$-correspondence $E$ over a $C^*$-algebra $A$ is a sort of generalized Hilbert space, together with a homomorphism $\phi : A \to \mathcal{L}(E)$ giving a left action of $A$ on $E$, where $\mathcal{L}(E)$ denotes the $C^*$-algebra of bounded adjointable operators on $E$. A generalized transfer operator for $\phi$ is a completely positive linear map $\tau : \mathcal{L}(E) \to A$ with the property that $\tau(\phi(a)X) = a\tau(X)$ for $a \in A$ and $X \in \mathcal{L}(E)$. Assuming $\phi$ has a generalized transfer operator, we give a way of constructing a unique coisometric extension for any so-called completely contractive covariant representation of $(E, A)$. Completely contractive covariant representations are generalizations of contractive operators to the setting of $C^*$-correspondences, and in the course of the construction we will examine several examples. Notable among them is the special case where $E = \mathbb{C}^d$ and $A = \mathbb{C}$, in which case a (unital) generalized transfer operator is simply a state on $M_d(\mathbb{C})$.

Our analysis extends work of Muhly and Solel and complements studies by Ball and Vinnikov, Bratteli and Jorgensen, Exel, and Popescu. (Received September 23, 2012)