Nevanlinna’s Representation. Let $\Pi = \{ z \in \mathbb{C} \mid \text{Im } z > 0 \}$ and let $h$ be a complex valued function defined on $\Pi$. There exists a finite positive Borel measure $\mu$ that is supported in $\mathbb{R}$ and such that

$$h(z) = \int \frac{1}{t - z} \, d\mu$$

for all $z \in \Pi$ if and only if $h$ is analytic on $\Pi$, $\text{Im } h(z) > 0$ for all $z \in \Pi$, and

$$\lim \inf_{y \to \infty} y|h(iy)| < \infty.$$

We shall describe how this theorem and other closely related theorems can be generalized to several variables using operator-theoretic methods. The results are drawn from a number of papers written jointly with Ryan Tully-Doyle, John McCarthy and Nicholas Young. (Received September 24, 2012)