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Let $S(t)$, defined for small $t > 0$, be a monotone family (i.e., $S(t) \subset S(u)$ for $t > u > 0$) of compact subsets of a compact set $K \subset \mathbb{R}^n$. Both $S(t)$ and $K$ are assumed to be “tame” (e.g., real semialgebraic). The following problem emerges from a conjecture formulated by Gabrielov and Vorobjov (2009) in connection with their work on approximation of a tame set by homotopy equivalent compact sets: Construct a triangulation of $K$ so that restriction of $S(t)$ to each open simplex is equivalent to a “standard” family. The explicit list of $2^n + 1$ standard families is based on lex-monotone Boolean functions in $n$ Boolean variables. A second conjecture claims that $K$ admits a cylindrical cell decomposition into regular cells, such that restriction of $S(t)$ to each open $k$-cell $C$ is either empty or a regular $k$-cell, and its boundary in $C$ is either empty or a regular $(k - 1)$-cell. To prove these conjectures, Basu, Gabrielov and Vorobjov introduced semi-monotone sets, a generalization of convex sets, and monotone maps, a multivariate generalization of univariate monotone functions. Recent progress towards the proof of the above conjectures will be reported. (Received September 19, 2012)