We will describe some results on the spectral theory of the Laplace-Beltrami operator on a compact Riemannian orbifold and its connections with dynamics of the geodesic flow. We extend classical results on quantum ergodicity due to Shnirelman, Colin de Verdiere and Zelditch to orbifolds, proving that the ergodicity of the geodesic flow on a compact Riemannian orbifold implies quantum ergodicity for the Laplace-Beltrami operator. For the proof of this theorem, we state two results, which maybe of independent interest. The first result is the local Weyl law for elliptic operators on orbifolds. The second result is a Egorov type theorem for orbifolds. To provide examples of ergodic geodesic flows on orbifolds, we prove the orbifold version of a classical result by Anosov, saying that the geodesic flow on a compact Riemannian orbifold of negative sectional curvature is ergodic. Finally, we will discuss some aspects of noncommutative spectral geometry of orbifolds. (Received September 19, 2012)