In 1998 Simon Donaldson and Richard Thomas initiated Gauge Theory for higher dimensional manifolds of special holonomy. In particular they discussed and made several key observations on the possible linearization/Fredholm index theory and perturbations/transversality of the relevant moduli spaces. Since then they and others have followed up the work in more special situations. Going in a different direction, Gang Tian in 2000 introduced a very general anti-self dual equation for any higher dimensional manifolds and proved an important compactification theorem for the moduli space. He also proved that the blow-up locus has a nice calibrated geometric structure and the singularity set is relatively small in terms of Hausdorff measure. In this talk we will propose another Donaldson type theory on higher dimensional manifolds carrying co-dimension 4 foliations. We introduce the foliated anti-self dual equation, which roughly can be treated as a special case of Tian’s equation. We prove a complete perturbation/transversality theorem for the moduli space. We also show an improved compactification theorem, which states that the blow-up locus consists of compact leaves and the singularity set is empty. (Received September 24, 2012)