We introduce a variation of the classical Ricci flow equation that modifies the volume constraint of that equation to a scalar curvature constraint. The resulting equations are named the \textbf{conformal Ricci flow equations} because of the role that conformal geometry plays in constraining the scalar curvature. These new equations are given by

\[
\frac{\partial g}{\partial t} + 2(Ric(g) + \frac{1}{n}g) = -pg \\
R(g) = -1
\]

for a dynamically evolving metric \( g \) and a non-dynamical scalar field \( p \geq 0 \), named the \textbf{conformal pressure}. The conformal pressure serves as a Lagrange multiplier to conformally deform the flow so as to maintain the scalar curvature constraint. The conformal Ricci flow equations are analogous to the \textbf{Navier-Stokes equations} of fluid mechanics

\[
\frac{\partial v}{\partial t} + \nabla v + \nu \Delta v = -\text{grad} \ p \\
\text{div} \ v = 0
\]

The conformal Ricci flow equations can be thought of as a Navier-Stokes equation for the metric \( g \), just as the classical Ricci flow equation can be thought of as a heat equation for \( g \). Properties of the conformal Ricci flow equations and their interpretation as a flow on the Teichmüller space of conformal structures are discussed. (Received September 26, 2012)